I want to put this in LaTeX eventually, but my comp is broken. I figured we should start to figure out terminology and formalize our thinking.

Definition 1: A **board** is a map from the integer lattice to the natural numbers (B \in Hom(\mathbb{Z} \times \mathbb{Z}, \mathbb{N})

The inputs represent the coordinates on the integer lattice that the lower left hand corner of the square occupies and the value refers to the number of chips on that square.

A **square** is an element of $\mathbb{Z} \times \mathbb{Z}$

Definition 2: We say that a board is **stable** if its values are strictly less than 4

Definition 3: We define the **Taxicab distance** between two squares $A=(x,y)$ and $B=(u,v)$ as

\begin{equation}

d(A,B)=|x-u| + |y-v|.

\end{equation}

Definition 4: We define the **big earthquake operator** as a map from the set of all boards to the set of all boards ( BE: Boards \rightarrow Boards), that transforms all of the squares on the board with the following rule:

\begin{equation}

BE(f(x,y)) = f(x,y) - 4\*I\_{f(x,y) \geq 4} + I\_{f(x-1,y) \geq 4} + I\_{f(x+1,y) \geq 4} + I\_{f(x,y-1) \geq 4} + I\_{f(x,y+1) \geq 4},

\end{equation}

where I\_{A \geq 4} is defined as

\begin{equation}

I\_{A \geq 4} =

\begin{cases}

1 \text{ if } A \geq 4 \\

0 \text{ if } A < 4

\end{cases}.

\end{equation}

Definition 5: We define the **small earthquake operator** as a map from the set of all boards to the set of all boards (SE: Boards \rightarrow Boards), that selects one square $(x,y)$ with $f(x,y) \geq 4$ and transforms it and all of its neighbors with the following rule:

\begin{align\*}

SE( f(x,y)) = f(x,y)-4 \\

SE( f(x+1,y)) = f(x+1,y)+1 \\

SE( f(x-1,y)) = f(x-1,y)+1 \\

SE( f(x,y+1)) = f(x,y+1)+1 \\

SE( f(x,y-1)) = f(x,y-1)+1,

\end{align\*}

and leaves all other squares unchanged.

Trivial Lemma: If a board is stable it is invariant under both earthquake operators

Definition 6: We define a “potential” function from the set of boards to the natural numbers ( F: \{ B \} \rightarrow \mathbb{N} ) by

\begin{equation\*}

F(B) = \sum\_{ i, j \in \mathbb[Z} \times \mathbb{Z} } B(i,j) \* ( |i| + |j| )^2

\end{equation\*}

Lemma: If a board $B$ is not stable then F(B) \leq F(SE(B)) +4 \leq F(BE(B))